BAC VARĂ 2025
TEHNOLOGIC

Subiectul I.

1. $\frac{1}{10}+3\cdot\left(\frac{^{5)}1}{2}-\frac{^{2)}1}{5}\right)=1$

$\frac{1}{10}+3\cdot\left(\frac{5}{10}-\frac{2}{10}\right)=1$

$\frac{1}{10}+3\cdot\frac{3}{10}=1$

$\frac{1}{10}+\frac{9}{10}=1$

$\frac{10}{10}=1$

$1=1$ (A)

2. $f(x)=6x-3$

$f(2)=a+f(0)$

$f(2)=6\cdot2-3=12-3=9$

$f(0)=6\cdot0-3=0-3=-3$

$\Rightarrow 9=a+(-3)$

$9=a-3 \Rightarrow a-3=9$

$a=9+3$

$a=12$

3. $5^{2x}=5^{3-x} \Rightarrow$

$2x=3-x$

$2x+x=3$

$3x=3 \quad |:3$

$x=1$

4. $A=\{0,1,2,3,4,\underbrace{5,6,7,8,9}_{\text{caz fav}}\}$

$6m>25$

cazuri pos $=10$

cazuri fav $=5$

$P=\frac{\text{caz fav}}{\text{caz pos}}=\frac{5^{(5}}{10}=\frac{1}{2}$

5. $A(2,0), B(2,1), C(6,4)$

$BC=\sqrt{(x_C-x_B)^2+(y_C-y_B)^2}=\sqrt{(6-2)^2+(4-1)^2}$

$BC=\sqrt{4^2+3^2}=\sqrt{16+9}=\sqrt{25}=5$

$AB=\sqrt{(x_B-x_A)^2+(y_B-y_A)^2}=\sqrt{(2-2)^2+(1-0)^2}$

$AB=\sqrt{0^2+1^2}=\sqrt{0+1}=\sqrt{1}=1$

$BC=a\cdot AB \Leftrightarrow 5=a\cdot 1 \Rightarrow a=5$

6. ΔMNP

$m(\sphericalangle M)=90^\circ$

$\left.\begin{array}{l} MN=4\cdot MP \\ MN=8 \end{array}\right| \Rightarrow 8=4\cdot MP \Rightarrow MP=\frac{8}{4} \Rightarrow MP=2$

$A_{\Delta dr}=\frac{c_1\cdot c_2}{2}=\frac{MN\cdot MP}{2}=\frac{8\cdot\cancel{2}}{\cancel{2}}=8$

Subiectul II.

1. $A=\begin{pmatrix}5 & 2\\ 2 & 1\end{pmatrix}, B=\begin{pmatrix}5 & 3\\ 3 & -1\end{pmatrix}, I_2=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

a) $\det A=1$

$\det A=\begin{vmatrix}5 & 2\\ 2 & 1\end{vmatrix}=5\cdot 1-2\cdot 2=5-4=1$

x	$-\infty$		-3		1		$+\infty$
ec.	+ + + + +		0	- - - - - - -	0	+ + + + + +	

$\Rightarrow x \in [-3, 1]$

2. $f = x^3 - 3x^2 + x + m$

a) $m = 1$, $f(1) = 0$

pt $m = 1 \Rightarrow f = x^3 - 3x^2 + x + 1$

$f(1) = 1^3 - 3 \cdot 1^2 + 1 + 1 = 1 - 3 + 2 = 3 - 3 = 0$

b) $S_1 = x_1 + x_2 + x_3 = -\frac{b}{a} = -\frac{-3}{1} = 3$

$S_2 = x_1x_2 + x_1x_3 + x_2x_3 = \frac{c}{a} = \frac{1}{1} = 1$

$S_3 = x_1x_2x_3 = -\frac{d}{a} = -\frac{m}{1} = -m$

$f = x^3 - 3x^2 + x + m$

$a = 1$, $b = -3$, $c = 1$, $d = m$

$2x_1 + 2x_2 + 2x_3 = 1 + x_1x_2x_3$

$2(x_1 + x_2 + x_3) = 1 + (-m)$

$2 \cdot 3 = 1 - m$

$6 = 1 - m$

$m = 1 - 6$

$m = -5$

c) $f : (x - 2) \Rightarrow r = -5 \Rightarrow f(2) = -5$

$f(2) = 2^3 - 3 \cdot 2^2 + 2 + m = 8 - 12 + 2 + m = -2 + m$

$\Rightarrow -2 + m = -5$

$m = -5 + 2 \Rightarrow m = -3$

b) $3A - 5I_2 = xB \quad , \quad x = ?$

$$3 \cdot \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix} - 5 \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = x \cdot \begin{pmatrix} 5 & 3 \\ 3 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 15 & 6 \\ 6 & 3 \end{pmatrix} - \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} = \begin{pmatrix} 5x & 3x \\ 3x & -x \end{pmatrix}$$

$$\begin{pmatrix} 10 & 6 \\ 6 & -2 \end{pmatrix} = \begin{pmatrix} 5x & 3x \\ 3x & -x \end{pmatrix}$$

$\Rightarrow 5x = 10 \quad | : 5$

$x = 2$

c) $\det(A \cdot (B - A) + xI_2) \leq 2$

$$B - A = \begin{pmatrix} 5 & 3 \\ 3 & -1 \end{pmatrix} - \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix}$$

$$A \cdot (B - A) = \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix} =$$

$$\begin{pmatrix} 5 \cdot 0 + 2 \cdot 1 & 5 \cdot 1 + 2 \cdot (-2) \\ 2 \cdot 0 + 1 \cdot 1 & 2 \cdot 1 + 1 \cdot (-2) \end{pmatrix} = \begin{pmatrix} 2 & 5 - 4 \\ 1 & 2 - 2 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$$

$$x \cdot I_2 = x \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix}$$

$$A \cdot (B - A) + xI_2 = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix} = \begin{pmatrix} 2 + x & 1 \\ 1 & x \end{pmatrix}$$

$$\det = \begin{vmatrix} 2 + x & 1 \\ 1 & x \end{vmatrix} = (2 + x) \cdot x - 1 \cdot 1 = 2x + x^2 - 1 = x^2 + 2x - 1$$

$$x^2 + 2x - 1 \leq 2 \Rightarrow x^2 + 2x - 1 - 2 \leq 0$$

$$x^2 + 2x - 3 \leq 0$$

$x^2 + 2x - 3 = 0$

$\Delta = b^2 - 4ac = 2^2 - 4 \cdot 1 \cdot (-3) = 4 + 12 = 16$

$$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-2 \pm \sqrt{16}}{2} = \frac{-2 \pm 4}{2}$$

$$\Rightarrow x_1 = \frac{-2 - 4}{2} = \frac{-6}{2} = -3 \qquad x_2 = \frac{-2 + 4}{2} = \frac{2}{2} = 1$$

$\Rightarrow f = x^3 - 3x^2 + x - 3 = x^2(x-3) + (x-3) = (x-3)(x^2+1)$

$\Rightarrow f \vdots (x^2+1)$

Subiectul III.

1. $f: \mathbb{R} \to \mathbb{R}$, $f(x) = \frac{x-2}{e^x}$

a) $f'(x) = \frac{3-x}{e^x}$

$f'(x) = \left(\frac{x-2}{e^x}\right)' = \frac{(x-2)' e^x - (x-2) \cdot (e^x)'}{(e^x)^2} =$

$= \frac{e^x - (x-2) \cdot e^x}{e^{2x}} = \frac{\cancel{e^x}(1-x+2)}{e^{\cancel{2x}}} = \frac{3-x}{e^x}$

b) $tg: y - f(x_0) = f'(x_0)(x-x_0)$, $x_0 = 0$

$tg: y - f(0) = f'(0)(x-0)$

$f(0) = \frac{0-2}{e^0} = \frac{-2}{1} = -2$ $\quad f'(0) = \frac{3-0}{e^0} = \frac{3}{1} = 3$

$tg: y - (-2) = 3 \cdot x$

$tg: y + 2 = 3x$

$tg: y = 3x - 2$

c) $f'(x) = 0 \Rightarrow \frac{3-x}{e^x} = 0 \Rightarrow 3 - x = 0 \Rightarrow x = 3$

x	$-\infty$		1		3		4		$+\infty$
$f'(x)$	+	+	+	+	0	−	−	−	−
$f(x)$									

pt $x \in [1,3]$, $f'(x) \geq 0 \Rightarrow f$ este crescătoare pe $[1,3]$

$\Rightarrow f(1) \leq f(x)$ $\quad (1)$

pt $x \in [3,4]$, $f'(x) \leq 0 \Rightarrow f$ este descrescătoare

pe $[3,4] \Rightarrow f(x) \leq f(3)$ (2)

din 1 și 2 $\Rightarrow f(1) \leq f(x) \leq f(3)$

$$f(1) = \frac{1-2}{e^1} = -\frac{1}{e} \qquad f(3) = \frac{3-2}{e^3} = \frac{1}{e^3}$$

$$\Rightarrow -\frac{1}{e} \leq \frac{x-2}{e^x} \leq \frac{1}{e^3} \quad | \cdot e^x$$

$$-e^{-1} \cdot e^x \leq x-2 \leq e^{-3} \cdot e^x$$

$$-e^{x-1} \leq x-2 \leq e^{x-3}$$

2. $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x^2+4x+8$

a) $$\int_0^1 (f(x)-x^2-8)\,dx = \int_0^1 (\cancel{x^2}+4x+\cancel{8}-\cancel{x^2}-\cancel{8})\,dx =$$

$$\int_0^1 4x\,dx = 4\int_0^1 x\,dx = \overset{2}{\cancel{4}} \cdot \frac{x^2}{\cancel{2}}\Big|_0^1 = 2 \cdot (1^2-0^2) = 2 \cdot 1 = 2$$

b) $$\int_0^8 \frac{x}{f(x)-4x}\,dx = \int_0^8 \frac{x}{x^2+\cancel{4x}+8-\cancel{4x}}\,dx = \int_0^8 \frac{x}{x^2+8}\,dx =$$

$t = x^2+8$

$dt = 2x\,dx \Rightarrow dx = \frac{dt}{2x}$

$$\Rightarrow \int_0^8 \frac{\cancel{x}}{t} \cdot \frac{dt}{2\cancel{x}} = \int_0^8 \frac{1}{t} \cdot \frac{dt}{2} = \frac{1}{2}\int_0^8 \frac{1}{t}\,dt = \frac{1}{2} \cdot \ln t \Big|_0^8$$

$$= \frac{1}{2} \cdot \ln(x^2+8)\Big|_0^8 = \frac{1}{2} \cdot (\ln(8^2+8) - \ln(0^2+8)) =$$

$$= \frac{1}{2} \cdot (\ln 72 - \ln 8) = \frac{1}{2} \cdot \ln \frac{72}{8} = \frac{1}{2} \cdot \ln 9 = \frac{1}{\cancel{2}} \cdot \cancel{2} \cdot \ln 3$$

$$= \ln 3$$

c) $\int_0^a \frac{1}{f(x)-4}\,dx = \frac{1}{4}$

$$\int_0^a \frac{1}{x^2+4x+8-4}\,dx = \int_0^a \frac{1}{x^2+4x+4}\,dx = \int_0^a \frac{1}{(x+2)^2}\,dx$$

$t = x+2$

$dt = dx$

$$\Rightarrow \int_0^a \frac{1}{t^2}\,dt = \int_0^a t^{-2}\,dt = \frac{t^{-2+1}}{-2+1}\Big|_0^a = \frac{t^{-1}}{-1}\Big|_0^a =$$

$$= -\frac{1}{t}\Big|_0^a = \cancel{-\left(\frac{1}{a}-\frac{1}{0}\right)=}$$

$$= -\frac{1}{x+2}\Big|_0^a = -\left(\frac{1}{a+2}-\frac{1}{0+2}\right) = -\frac{1}{a+2}+\frac{1}{2}$$

$$\Rightarrow -\frac{1}{a+2}+\frac{1}{2} = \frac{1}{4}$$

$$-\frac{1}{a+2} = \frac{1}{4} - {}^{2)}\frac{1}{2}$$

$$-\frac{1}{a+2} = \frac{1}{4} - \frac{2}{4}$$

$$-\frac{1}{a+2} = -\frac{1}{4} \quad | \cdot (-1)$$

$$\frac{1}{a+2} = \frac{1}{4} \Rightarrow a+2 = 4$$

$$a = 4-2$$

$$a = 2$$